ECE 443 HW 5

November 21, 2011

1. Let $x(t) = 30\cos(10^6 t + 4\sin(15t))$ be the input to a $5\times$ frequency multiplier with output $y(t)$.

   (a) Find the instantaneous frequency and estimated bandwidth of $x(t)$.

   (b) Find the instantaneous frequency and estimated bandwidth of $y(t)$.

   (c) If frequency modulation is used, what is the modulating signal ($k_f m(t)$)?

   (d) If phase modulation is used, what is the modulating signal ($k_p m(t)$)?

   (e) Find the power of $x(t)$.

2. Let $m(t)$ be a signal bandlimited to 20 kHz. $m(t)$ is used to modulate the FM signal $x_1(t)$ with $\Delta f = 500$kHz and a carrier frequency of 3MHz. x_2 is the output of a $7\times$ frequency multiplier (whose input is x_1), and x_3 is the output of an $11\times$ frequency multiplier (whose input is x_2).

   (a) Find the center frequencies f_{c1}, f_{c2}, and f_{c3}.

   (b) Find the peak frequency deviations Δf_1, Δf_2, and Δf_3.

   (c) Estimate the bandwidths of x_1, x_2, and x_3.

3. A low power FM exciter is available which can produce an FM signal with $f_\Delta = 10$kHz and a carrier frequency of 8 MHz. Using this exciter, design a transmitter which will generate an FM signal with peak frequency deviation of 100 kHz and a carrier frequency of 60 MHz. You can use blocks like mixers, oscillators, filters, summers, frequency multipliers, etc.

4. A message signal has a bandwidth $W = 15$kHz. Estimate the FM transmission bandwidth for $f_\Delta = 0.1$, 0.5, 1, 5, 10, 50, 100, and 500 kHz.

5. An FM radio station alternates between music and talk programming. The music is bandlimited to $W = 15$ kHz and talk is bandlimited to 5 kHz. Assuming $f_\Delta/W = 5$ for both music and talk, what percentage of maximum music bandwidth is used by the talk program?

6. The covariance of two random variables X and Y is $\mathrm{Cov}(X,Y) = E\{(X-\mu_X)(Y-\mu_Y)\}$ (to be clear, the expectation is over both X and Y). Expand this expectation and simplify it when:

   (a) X and Y are independent.

   (b) X and Y are related by $Y = \alpha X + \beta$, where α and β are constants.

7. *(ECE644 only)* Let X and Y be a random variables, and let $\tilde{Y} = \alpha X + \beta$ be a linear estimate of Y. Find expressions for α and β to minimize the mean squared error $\epsilon^2 = E\{(Y-\tilde{Y})^2\}$. *Hint: Expand ϵ^2 into a quadratic polynomial of α and β. You can then find the minimum with standard calculus techniques.*